

Automatic CP Invariance and Flavor Symmetry

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Abstract

The approximate conservation of CP can be naturally understood if it arises as an automatic symmetry of the renormalizable Lagrangian. We present a specific realistic example with this feature. In this example, the global Peccei-Quinn symmetry and gauge symmetries of the model make the renormalizable Lagrangian CP invariant but allow non zero hierarchical masses and mixing among the three generations. The left-right and a horizontal $U(1)_H$ symmetry is imposed to achieve this. The non-renormalizable interactions invariant under these symmetries violate CP whose magnitude can be in the experimentally required range if $U(1)_H$ is broken at very high, typically, near the grand unification scale.

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The $SU(3) \otimes SU(2) \otimes U(1)$ symmetry associated with the Standard Model (SM) of electroweak interactions is known to be inadequate for explaining fermionic masses and mixing. The gauge symmetry of the SM can accommodate these masses and in particular the CP violation [1] but does not provide any theoretical understanding of mass hierarchy or of approximate CP conservation. Some understanding of these issues can be obtained by imposing additional symmetries acting in the space of fermionic flavors. Such horizontal symmetries are known [2, 3] to lead to desired patterns of fermionic masses and mixing. It can also help in understanding the approximate conservation of CP . The aim of this note is to discuss this aspect of horizontal symmetry through an example. In this example, the exact conservation of a horizontal $U(1)$ symmetry leads automatically to a CP conserving theory while its breakdown at very high scale leads to the observed CP violation.

Ideally one would like to have CP as an automatic symmetry of the renormalizable Lagrangian in analogy with the baryon and the lepton number symmetries which are consequences of the gauge structure and the field content in the standard model. This actually happens in a special case with two generations of fermions [1, 4]. In this case, the most general Lagrangian invariant under the SM interactions is automatically CP invariant if there is only one Higgs doublet or if there are two Higgs doublets but natural flavor conservation is imposed as an additional symmetry [4]. This feature however gets spoiled when one introduces the third generation.

In principle the presence of the third generation need not spoil the CP invariance if Yukawa couplings are suitably restricted. To be realistic, these restrictions must however be such that all masses and mixing angles are non-zero and hierarchical in accordance with the observed pattern. This can be accomplished if additional gauge interactions are postulated.

We will present an explicit example where the same horizontal symmetry gives Fritzsch structure [5] for the quark mass matrices and also leads automatically to a CP invariant Lagrangian. In realistic case, one needs CP violation as well as deviations from the Fritzsch structure [6]. Both these occur through non-renormalizable interactions when the horizontal symmetry is broken at very high scale. The smallness of CP violation in this case is thus intimately linked to the scale of horizontal symmetry breaking.

Our example requires extension of $SU(3) \otimes SU(2) \otimes U(1)$ to a left-right symmetric theory [7]. In addition to the $G_{LR} = SU(3) \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{(B-L)}$ group we need to impose a horizontal symmetry $U(1)_H$ and the Peccei-Quinn [8] (PQ) symmetry $U(1)_{PQ}$ in order to get a fully CP invariant theory.

The $U(1)_H$ is a gauged horizontal symmetry which is chosen to obtain texture zeroes in the quark mass matrices. The choice of $U(1)_H$ is constrained by the requirement of anomaly cancellation. Anomalies are seen to cancel if one chooses the $U(1)_H$ charges (1, 0, -1) for the left handed quark fields denoted in the weak basis by q'_{iL} . The corresponding right handed fields are chosen to have opposite $U(1)_H$ values. We need to introduce three bi-doublet Higgs fields Φ_α with the $U(1)_H$ charges (1, -1, -2). These Higgs fields are needed in order to obtain essentially real but non-trivial quark mass matrices with non-vanishing masses and mixing angles.

The $U(1)_{PQ}$ is a global Peccei Quinn symmetry which serves dual purpose here. It allows rotation of the strong CP violating angle θ [8] and it also forbids some crucial couplings in the Yukawa and Higgs sectors. Under the PQ symmetry, $q'_{iR} \rightarrow e^{i\beta} q'_{iR}$ and $\Phi_\alpha \rightarrow e^{-i\beta} \Phi_\alpha$. Rest of the fields remain invariant. Given this choice, the most general $G \equiv G_{LR} \otimes U(1)_{PQ} \otimes U(1)_H$ invariant Yukawa couplings can be written as

$$- \mathcal{L}_Y = \bar{q}'_L \Gamma_\alpha \Phi_\alpha q'_R + H.C. \quad (1)$$

with

$$\Gamma_1 = \begin{pmatrix} 0 & a & 0 \\ a^* & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \Gamma_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & b \\ 0 & b^* & 0 \end{pmatrix}; \Gamma_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & c \end{pmatrix}; \quad (2)$$

We have imposed here the conventional discrete parity [7] $q'_L \leftrightarrow q'_R$ and $\Phi_\alpha \leftrightarrow \Phi_\alpha^\dagger$. CP is not imposed as a symmetry and hence the couplings a, b appearing in Γ_α are complex in general. But their phases can be rotated away leaving a CP invariant Lagrangian. In order to show this, we first concentrate on the G invariant scalar potential for the fields Φ_α and $\tilde{\Phi}_\alpha = \tau_2 \Phi_\alpha^* \tau_2$:

$$\begin{aligned} V_1(\Phi) = & \mu_\alpha^2 tr(\Phi_\alpha^\dagger \Phi_\alpha) + \lambda_\alpha \{tr(\Phi_\alpha^\dagger \Phi_\alpha)\}^2 \\ & + \lambda_{1\alpha,\beta} tr(\Phi_\alpha^\dagger \tilde{\Phi}_\beta) tr(\tilde{\Phi}_\alpha^\dagger \Phi_\beta) \\ & + \rho_{1\alpha} tr(\Phi_\alpha \Phi_\alpha^\dagger \Phi_\alpha \Phi_\alpha^\dagger) + \rho_{2\alpha} tr(\Phi_\alpha^\dagger \tilde{\Phi}_\alpha \tilde{\Phi}_\alpha^\dagger \Phi_\alpha) + \rho_{3\alpha} tr(\Phi_\alpha \tilde{\Phi}_\alpha^\dagger \tilde{\Phi}_\alpha \Phi_\alpha^\dagger) \\ & + \sum_{\alpha \neq \beta} \left\{ \lambda_{2\alpha,\beta} tr(\Phi_\alpha^\dagger \Phi_\beta) tr(\Phi_\beta^\dagger \Phi_\alpha) + \lambda_{3\alpha,\beta} tr(\Phi_\alpha^\dagger \Phi_\alpha) tr(\Phi_\beta^\dagger \Phi_\beta) \right. \\ & + \delta_{1\alpha\beta} tr(\Phi_\alpha^\dagger \Phi_\beta \Phi_\beta^\dagger \Phi_\alpha) + \delta'_{1\alpha\beta} tr(\Phi_\beta^\dagger \Phi_\beta \Phi_\alpha^\dagger \Phi_\alpha) \\ & + \delta_{2\alpha\beta} tr(\Phi_\alpha^\dagger \tilde{\Phi}_\beta \tilde{\Phi}_\alpha^\dagger \Phi_\beta) + \delta'_{2\alpha\beta} tr(\tilde{\Phi}_\alpha^\dagger \tilde{\Phi}_\beta \Phi_\alpha^\dagger \Phi_\beta) \\ & \left. + \delta_{3\alpha\beta} tr(\Phi_\alpha \tilde{\Phi}_\beta^\dagger \tilde{\Phi}_\beta \Phi_\alpha^\dagger) + \delta'_{3\alpha\beta} tr(\Phi_\alpha \Phi_\alpha^\dagger \tilde{\Phi}_\beta \tilde{\Phi}_\beta^\dagger) \right\} \end{aligned} \quad (3)$$

The combined requirement of hermiticity and $U(1)_H \otimes U(1)_{PQ}$ symmetry forces all the parameters of $V_1(\Phi)$ to be real [9]. As a consequence, CP appears as a symmetry of $V_1(\Phi)$ although this was not imposed. One could choose a CP conserving minimum for a suitable range of parameters :

$$\langle \Phi_\alpha \rangle \equiv \begin{bmatrix} \kappa_{\alpha u} & 0 \\ 0 & \kappa_{\alpha d} \end{bmatrix} \quad (4)$$

where $\kappa_{\alpha u}$ and $\kappa_{\alpha d}$ are real. Eqs.(2) and (4) imply the following quark mass matrices:

$$M_{u,d} = \begin{bmatrix} 0 & a\kappa_{1u,d} & 0 \\ a^*\kappa_{1u,d} & 0 & b\kappa_{2u,d} \\ 0 & b^*\kappa_{2u,d} & c\kappa_{3u,d} \end{bmatrix} \quad (5)$$

Note that the M_u and M_d allow for general up and down quark masses in spite of the correlated structures. However because of this correlation, M_u and M_d can be simultaneously made real with a diagonal phase matrix P :

$$\widehat{M}_{u,d} \equiv PM_{u,d}P^\dagger = \begin{bmatrix} 0 & |a|\kappa_{1u,d} & 0 \\ |a|\kappa_{1u,d} & 0 & |b|\kappa_{2u,d} \\ 0 & |b|\kappa_{2u,d} & |c|\kappa_{3u,d} \end{bmatrix} \quad (6)$$

Phases in P can be easily related to that in a and b . $\widehat{M}_{u,d}$ are diagonalised by orthogonal matrices

$$O_{u,d}\widehat{M}_{u,d}O_{u,d}^T = \text{diag}(m_{iu,d})$$

Let us now discuss the CP properties of the model. Because of the fact that both M_u and M_d can be made real by the same phase matrix P , the Kobayashi Maskawa matrices in the left as well as the right handed sectors are real. The reality of $\kappa_{\alpha u,d}$ also imply that the $W_L - W_R$ mixing is real. Hence gauge interactions are CP conserving. Moreover the matrix P appearing in eq.(6) in fact make the individual Yukawa couplings real, i.e.

$$P\Gamma_\alpha P^\dagger = |\Gamma_\alpha| \quad (7)$$

for every α . This has the consequence that the couplings of the neutral and charged Higgses to the mass eigenstates of quarks also become real. As a result, the Higgs interactions would also conserve CP as long as mixing among the Higgs fields is CP conserving. This is assured by the CP invariance of $V_1(\Phi)$ and reality of $\langle \Phi_\alpha \rangle$. It follows from the above arguments that the model presented so far is in fact CP conserving although one did not impose it anywhere.

We have not yet introduced fields needed to break $SU(2)_R \otimes U(1)_{PQ} \otimes U(1)_H$. This can be done without spoiling the automatic CP invariance obtained above. As a concrete

example let us introduce the conventional [7] $SU(2)$ triplet Higgses $\Delta_{L,R}$ with zero $U(1)_H$ and $U(1)_{PQ}$ charges. The breaking of the PQ symmetry by $\langle \Phi_\alpha \rangle$ generates a weak scale axion. We need to introduce a $G_{L,R} = SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ singlet σ in order to make this axion invisible [10]. σ is taken to transform under PQ symmetry as $\sigma \rightarrow e^{-i\beta}\sigma$ and remains invariant under $U(1)_H$. Finally, we introduce a $G_{L,R}$ singlet field η_H with $U(1)_H$ charge -2 and transforming under the PQ symmetry as $\eta_H \rightarrow e^{-2i\beta}\eta_H$. The most general Higgs potential involving these fields and their couplings to Φ fields can be written as:

$$V_2 = \mu_{22}tr(\Phi_2\tilde{\Phi}_2^\dagger)\eta_H^* + \delta_{12}tr(\Phi_1\tilde{\Phi}_2^\dagger)\sigma^{*2} + \text{H.c} \\ + V(\Delta) + V(\Delta-\Phi) + V(\eta_H-\sigma-\Delta-\Phi)$$

For brevity, we do not display the parts $V(\Delta)$, $V(\Delta-\Phi)$ and $V(\eta_H-\sigma-\Delta-\Phi)$ but mention that they contain only real couplings [11]. The only complex couplings possible are μ_{22} and δ_{12} . But their phases can be absorbed into redefining σ and η_H without effecting reality of other parameters in V_2 . Thus the above V_2 is automatically CP conserving just like V_1 of eq.(3). V_1 and V_2 together constitute the complete scalar potential of the model.

We had imposed the discrete parity in the above analysis in order to obtain the Fritzsch textures for $M_{u,d}$. But the automatic CP invariance follows even in more general situation without the discrete parity. In this case, M_u and M_d are no longer hermitian, but $U(1)_H$ symmetry still preserves texture zeroes appearing in (13), (31), (11) and (22) elements of M_u and M_d . It can be shown [11] that even in this more general situation, the above argument goes through and one obtains automatic CP invariance. In contrast to the discrete parity, the left-right symmetry plays a crucial role in giving the correlated structures for $M_{u,d}$ which lead to a CP invariant theory.

Having presented a CP invariant theory, we now discuss possible ways which lead to small departures from exact CP invariance. Obvious way is to enlarge the Higgs sector in such a way that CP gets violated in mixing among the Higgs scalars. Alternative possibility is to assume that the horizontal symmetry gets broken at a very high scale viz. grand unification scale. In this case [3] the G invariant non-renormalizable couplings can induce sizable Yukawa coupling at the low scales. This possibility is discussed by many authors [3] with a view of understanding the textures of the fermion masses. In the present context, such terms would also induce naturally small CP violation. In fact the model presented

above allows the following general dim-5 terms resulting in fermion masses:

$$-\mathcal{L}_{NR} = \frac{1}{M} \bar{q}_L \Gamma'_\alpha \tilde{\Phi}_\alpha q_R \eta_H + \text{H.C.} \quad (8)$$

Here M is some heavy mass scale which we take to be the Planck scale M_P . The textures for Γ'_α are dictated by the $U(1)_H$ symmetry. The contribution of \mathcal{L}_{NR} to quark masses depends upon the parameter $\epsilon \equiv \frac{\langle \eta_H \rangle}{M_P}$.

The M_u and M_d following from eqs. (5) and (8) can be written as [12] :

$$M_{u,d} = \begin{bmatrix} 0 & a\kappa_{1u,d} & \epsilon\kappa_{3d,u}(\Gamma'_3)_{13} \\ a^*\kappa_{1u,d} & \epsilon\kappa_{3d,u}(\Gamma'_3)_{22} & b\kappa_{2u,d} + \epsilon\kappa_{2d,u}(\Gamma'_2)_{23} \\ \epsilon\kappa_{3d,u}(\Gamma'_3)^*_{13} & b^*\kappa_{2u,d} + \epsilon\kappa_{2d,u}(\Gamma'_2)^*_{23} & c\kappa_{3u,d} \end{bmatrix}$$

The non-renormalizable contribution signified by ϵ works in a dual way here. Firstly the presence of ϵ no longer makes it possible to rotate away the phase from $M_{u,d}$ and hence from the KM matrix. Secondly it also modifies the Fritzsch texture obtained in the above example. This is a welcome feature in view of the fact that the Fritzsch ansatz is found to be inconsistent [6] with the large top mass. The texture of $M_{u,d}$ obtained above retains the successful predictions of the original ansatz and is also consistent phenomenologically.

Note that the original Fritzsch ansatz implies that in the limit $\epsilon \rightarrow 0$,

$$|a\kappa_{1u}| \sim \sqrt{m_u m_c}; \quad |b\kappa_{2u}| \sim \sqrt{m_c m_t}; \quad |c\kappa_{3u}| \sim m_t;$$

$$|a\kappa_{1d}| \sim \sqrt{m_d m_s}; \quad |b\kappa_{2d}| \sim \sqrt{m_s m_b}; \quad |c\kappa_{3d}| \sim m_b$$

It follows therefore that $|\kappa_{2,3d}| \ll |\kappa_{2,3u}|$. Hence the presence of ϵ terms alters the structure of M_d more significantly than that of M_u . To a good approximation [11] one may take M_u as in eq.(6) and M_d as follows

$$M_d \sim \begin{bmatrix} 0 & |a|\kappa_{1d} & \epsilon\kappa_{3u}\delta_1 e^{i\alpha} \\ |a|\kappa_{1d} & \epsilon\kappa_{3u}\delta_2 & |b|\kappa_{2d} \\ \epsilon\kappa_{3u}\delta_1 e^{-i\alpha} & |b|\kappa_{2d} & c\kappa_{3d} \end{bmatrix} \quad (9)$$

As before, we have redefined the quark fields and absorbed the phases of (12) and (23) elements. But this now leaves phases in terms involving ϵ .

Since the matrix diagonalising M_u is completely fixed in terms of up-quark masses, we can express M_d of eq.(9) in terms of the known parameters as

$$M_d = O_u^T \mathcal{K} \text{diag}(m_d, -m_s, m_b) \mathcal{K}^\dagger O_u$$

where \mathcal{K} is the KM matrix in the Wolfenstein parameterization [13]. Comparing above M_d with the R.H.S of eq.(9) implies the successful relation

$$\lambda = \sqrt{\frac{m_d}{m_s}} - \sqrt{\frac{m_u}{m_c}}$$

Moreover the other parameters also get fixed in terms of the masses and mixing angles. Specifically,

$$\begin{aligned} a\kappa_{1d} &\approx -\sqrt{m_d m_s} \ ; \ b\kappa_{2d} \approx -m_b \lambda^2 \left(A + \frac{1}{\lambda^2} \sqrt{\frac{m_c}{m_t}} \right) \ ; \ c\kappa_{3d} \approx m_b; \\ \epsilon\kappa_{3u}\delta_2 &\approx -m_s(1 - \lambda^2) + m_b \left(\lambda^2 A + \sqrt{\frac{m_c}{m_t}} \right)^2 \ ; \\ \epsilon\kappa_{3u}\delta_1 \cos \alpha &\approx m_b A \lambda^3 \left(\rho - \frac{1}{\lambda} \sqrt{\frac{m_u}{m_c}} \right) \ ; \ \epsilon\kappa_{3u}\delta_1 \sin \alpha \approx m_b A \lambda^3 \eta \end{aligned}$$

where A , ρ and η are parameters in Wolfenstein matrix [13]. The exact value of ϵ depends upon other parameters. If one chooses Yukawa couplings $c, \delta_2 \sim O(1)$ then $\epsilon \sim \frac{m_s}{m_t} \sim 10^{-3}$. Consistency then requires $\delta_1 \sim 10^{-2}$ in this case. For $\epsilon \sim 10^{-3}$, the $U(1)_H$ symmetry breaking scale is required to be of the order of 10^{16} GeV [14] if the scale of the non-renormalizable terms is set by the Planck mass.

In summary we have discussed one possible approach to understanding of small CP violation in this paper. This is intimately linked to recent approaches which try to understand the fermionic mass textures through higher dimensional terms generated by flavor symmetry breaking at very large scale. This introduces in the low energy theory an effective small parameter controlling CP violation. The horizontal symmetry cannot be directly probed in this case. Alternative possibility not discussed here is to assume that horizontal symmetry is broken at low \sim TeV scale. In this case, CP violation can be introduced [11] through enlargement in the Higgs sector. In either case, the conservation of CP gets linked intimately to the horizontal symmetry.

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